EFFECTIVE MATERIAL PARAMETERS OF DIELECTRIC LAYER WITH METAL PERIODIC ARRAY

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Abstract The problem of effective material parameter deriving of homogeneous layer is considered to obtain exactly the same complex transmission and reflection coefficients of normal incident plane wave as ones are produced by metal periodic array placed on the surface of dielectric substrate.

Introduction

We offer physically consistent statement of effective material parameters of homogenous magnetic-dielectric layer modeled metal array on the surface of dielectric substrate to produce *exact* values of complex reflection and transmission coefficients. Unexpected result is an existence positive imaginary part of effective permittivity and permeability in resonance frequency region. This result does not contradict to energy conservation law. The reason is strong spatial dispersion of fields in periodic array structure. The appearance of positive imaginary part of effective permeability of periodic structure was mentioned also in [1]. Certainly the media with such peculiar material parameters are not realized by using any ordinary magnetic-dielectric. On the other hand their introducing in the formulas of layer reflection and transmission does not differ from using ordinary permittivity and permeability.

The models based on effective material parameters have to satisfy to the 'first physical principles' as the causality principle and the energy conservation law independently there is or absent the spatial dispersion [2]. However, at least two problems appear when effective material parameters are introduced for layer equivalent to resonance array on substrate. They are seen especially distinctly for lossless structure of array on the substrate. Actually, the permittivity and permeability of effective layer are complex values. As well known it is the consequence of the causality principle for any frequency dispersive medium. Thus the first problem is an appearance of complex material parameters for initially lossless structure. The second one is a fulfillment of energy conservation law for the model of sub-wavelength array with complex effective material parameters. The energy equality is violated in a number of existing models based on introducing effective parameters.

Our goal is the introducing of material parameters consistent with physical lows and producing complex reflection and transmission coefficients exactly corresponded to their values for initial structure. Below we consider this problem for array of C-shaped particles on the dielectric substrate.

The problem definition and solution method

An array of C-shaped particles is shown in Fig. 1(a). Elements of array are assumed perfect conducting and infinitely thin. Substrate is lossless or lossy dielectric with $\varepsilon_s = \varepsilon_s' - i\varepsilon_s''$ and $\mu_s = 1$. For the sake of simplicity we will consider normal incidence of unit amplitude plane wave polarized along y-axis from the region $z > 0$ only. We will treatment the above mentioned problem in the frequency range from 1 GHz up to 98 GHz. Free space wavelength λ is more than array period in this frequency band.

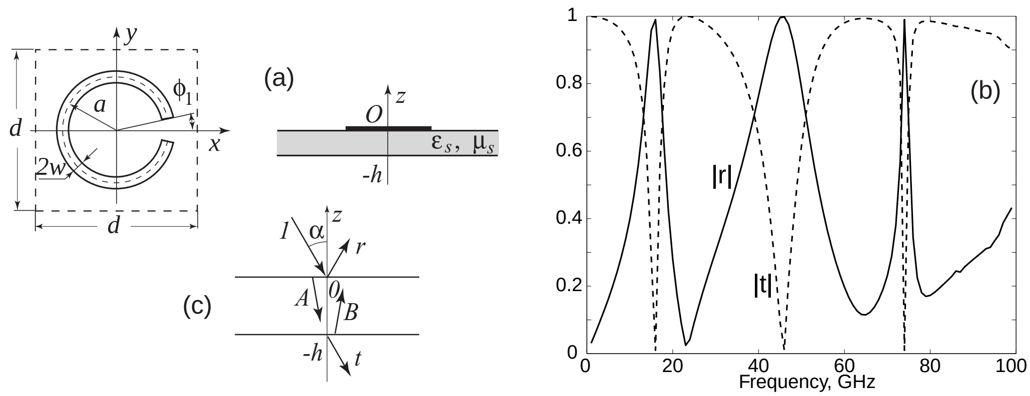

Fig. 1. (a) - Array of C-shaped strip particles: $d = 3$ mm, $a = 1.25$ mm, $\phi_1 = 10^\circ$, $w = 0.05$ mm, $\varepsilon_s = 3 - i\varepsilon_s''$, $\mu_s = 1$, $h = 0.25$ mm; (b) – Frequency dependencies of absolute values of the reflection and transmission coefficients for array with lossless substrate; (c) – Effective slab and field amplitudes in the incline incidence auxiliary problem.

Thus enough far from array the reflected and transmitted fields may be expressed in the form

$$E^r = r\exp(-ikz), \qquad E^t = t\exp(ik(z+h)), \tag{1}$$

where $k = \omega\sqrt{\varepsilon_0\mu_0}$, we assume the time dependence in the form $\exp(i\omega t)$. The coefficients of transmission t and reflection r were calculated from the numerical solution of the integral equation for the surface current on the strip particles by using the algorithm [3] based on the method of moments. The frequency dependencies of $|r|$ and $|t|$ are presented in Fig. 1(b).

Now we replace array on the substrate by homogeneous layer with material parameters $\varepsilon = \varepsilon' - i\varepsilon''$ and $\mu = \mu' - i\mu''$, and the same thickness h. Though final model will be created for normal incident wave, nevertheless the consideration is convenient to begin from expressions for reflection and transmission coefficients of incline incidence, Fig. 1(c). The point is that formulas for normal incidence are so ordinary to keep initial meaning and properties of complex variables. Thus we consider TE-polarized wave $E^i = \exp(-i\mathbf{k}^i\mathbf{r})$ incidence in the plane xOz on the effective slab. Reflected, transmitted and inner slab fields are following

$$E^r = r\exp(-i\mathbf{k}^r\mathbf{r}), \quad E^t = t\exp(-i\mathbf{k}^t(\mathbf{r}+\mathbf{e}_z h)), \quad E^{in} = A\exp(-i\mathbf{k}^-\mathbf{r}) + B\exp(-i\mathbf{k}^+(\mathbf{r}+\mathbf{e}_z h)), \tag{2}$$

where all wave vectors have the same projections on the x-axis equal to $k_t = k\sin\alpha$, $k_z^i = -k\cos\alpha$, $k_z^{\pm} = \pm\sqrt{k^2\varepsilon\mu - k_t^2}$. The waves (2) inside the slab may be decayed only. Thus from the principle of limit absorption a natural demand for the choice of square root branch is following. The root $\sqrt{\xi}$ has $\operatorname{Im}\sqrt{\xi} \le 0$ in complex plane ξ and if an equality fulfilled $\operatorname{Im}\sqrt{\xi} = 0$ that $\operatorname{Re}\xi \ge 0$.

Let us now present the expressions for transmission and reflection coefficients

$$t = \frac{4\mu k_z^+ k_z^i \exp(-ik_z^+ h)}{(k_z^+ + k_z^i\mu)^2 q - (k_z^+ - k_z^i\mu)^2}, \qquad r = \frac{[(k_z^+)^2 - (k_z^i)^2\mu^2](1-q)}{(k_z^+ + k_z^i\mu)^2 q - (k_z^+ - k_z^i\mu)^2}, \tag{3}$$

where $q = \exp(-2ik_z^+ h)$. For the normal incidence a conventional well known form of expressions (3) is

$$t = \left[\cos khn + \frac{i}{2}\left(Z + \frac{1}{Z}\right)\sin khn\right]^{-1}, \qquad r = \frac{i}{2}\left(Z - \frac{1}{Z}\right)t\sin khn. \tag{4}$$

In the expressions (4) the variables n and Z are introduced as $n = \lim(k_z^+/k) = \sqrt{\varepsilon\mu}$ and $Z = \lim(k\mu/k_z^+)$ at $\alpha \to 0$. Thus refractive index n can take on a value the same as defined above the square root only. However the complex parameter Z can take on *any value* in the complex plane because it was not need to involve any restrictions for complex values ε and μ. Now we can use known expressions

$$\cos khn = \frac{1 - r^2 + t^2}{2t}, \qquad Z = \pm\sqrt{\frac{(1+r)^2 - t^2}{(1-r)^2 - t^2}}. \tag{5}$$

They are derived from (4) rigorously. The difference of our approach with previous ones (for example [1, 4]) in this point is absent of any restrictions on a value Z in complex plane.

By using numerically found r and t we calculate from (5) value n beginning from the low frequency band boundary for guaranteeing an unambiguity. Then we find two possible values of Z from (5) and make choice of single one by testing fulfillment of (4). Effective permittivity and permeability of the slab are found from expressions $\varepsilon = n/Z$, $\mu = nZ$.

Discussion of numerical results

Calculated effective ε and μ lead to exact values of amplitudes and phases of r and t in considered frequency band for as lossless so as lossy substrate. Therefore energy conservation low $|r|^2 + |t|^2 = 1$ is fulfilled exactly for lossless array. The calculations were performed for a set of values $\varepsilon_s'' = 0$, 0.01, 0.05, 0.1. The resonance frequencies have approximately the same values as for array with lossless substrate so as for lossy ones. The lowest resonance frequency is 15.69 GHz for lossless structure and it is shifted to 15.68 GHz for $\varepsilon_s'' = 0.1$. The values of reflection coefficient $|r|$ are 1.0 and 0.93 and values of error of fulfillment energy conservation low are 10^{-9} and 0.13 correspondingly. In last case the difference is due to lossy substrate absorption. There are poles of effective permittivity in the frequencies of full reflection for lossless structure (see Fig. 2(a)). Because the function $\varepsilon(f)$ has infinitely large discontinuity the graphs are correct in all frequencies with the exception of small discontinuity neighborhoods. If the substrate of array is lossy, the poles disappear in the real frequencies. However variations are essential as real so as imagine parts of ε near resonance frequencies even for a sufficiently large $\varepsilon_s'' = 0.1$. Effective permittivity of lossy array is shown in Fig. 2(b) in the neighborhood of lowest frequency resonance. Frequency dependence of effective μ is shown in Fig. 3. There is not such essential influence of substrate loss on effective permeability as on the effective permittivity. If array is lossless the resonance reflection is the same as due to perfect conducting metal i.e. reflection coefficient is equal to -1. Because the magnetic field is equal to zero in

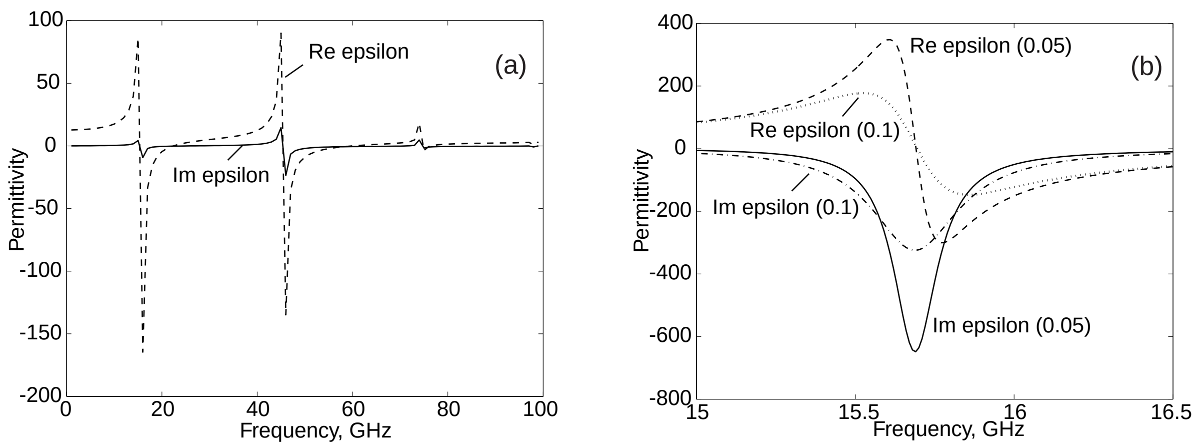

Fig. 2. Frequency dependence of effective permittivity of array placed on lossless substrate (a), and the same array on lossy substrate in frequency range of first resonance (b). The parenthesis numbers are corresponding to ε_s'' of the substrate.

perfect metal, effective permeability is zero in the resonance. Far from the resonances the value μ is close to unit due to absent magnetic properties of original materials of array and substrate. Thus effective μ is dispersive and complex consequently. Values ε'' and μ'' are associated commonly with the heat dissipation. Actually average power of heat losses is considered as $Q = (\omega/2)\int_V (\varepsilon_0 \varepsilon'' |E|^2 + \mu_0 \mu'' |H|^2) dV$.

However the value Q has rigorously single-valued meaning of heat in the media with small spatial dispersion only [2]. It means that the length of attenuation must be much more than wavelength and also much more than

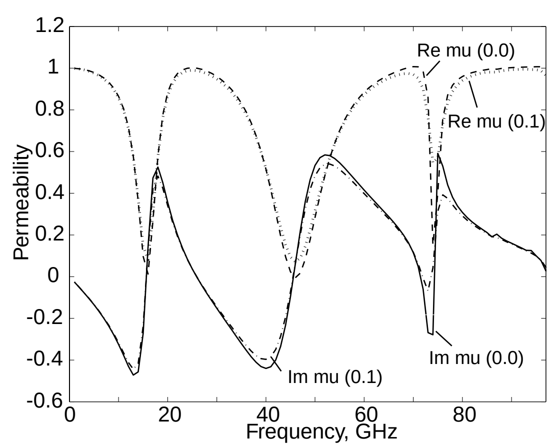

Fig. 3. Frequency dependence of effective permeability. The parenthesis numbers are corresponding to ε_s''.

linear size of considered volume. On the contrary a spatial dispersion is very strong in the slab with resonance array. Our numerical results show unambiguously fulfillment of energy equality for lossless structure. Detail study of frequency dependencies of effective ε and μ imaginary parts shows that ε'' and μ'' equal to zero simultaneously in the frequencies corresponded to full resonance reflection only for array with lossless substrate. In all rest frequencies either $\mathrm{Im}\,\varepsilon$ or $\mathrm{Im}\,\mu$ is positive. Thus lossless array cannot be simulated *exactly* by slab of conventional magnetic-dielectric in any frequencies besides a few points. The substrate loss does not modify essentially this conclusion.

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